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CALCULATION OF SHORT TERM AVERAGER OUTPUT STANDARD DEVIATION F0--ETC(U)
FEB 69 J P IANNIELLO
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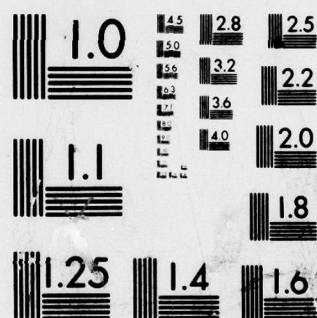
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CALCULATION OF SHORT TERM AVERAGER OUTPUT
STANDARD DEVIATION FOR A DIMUS SYSTEM

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by
10 John P. Ianniello

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I. INTRODUCTION

The digital number representing the Short Term Averager (STA) output of a DIMUS system must be requantized to fewer bits in order to limit the size of further averagers. It is desirable, however, to maintain linearity for small signal to noise ratios (reference (1)). A reasonable compromise between minimizing bit requirements and maintaining linearity is to requantize the STA data so that it is approximately linear in the range $\pm \sqrt{V_b}$ (where $\sqrt{V_b}$ is the STA output standard deviation for noise only at the hydrophones) and hard-limited for numbers outside this range. Therefore, it is necessary for design purposes to calculate $\sqrt{V_b}$. This number is usually found under the assumption that the time sampled inputs to the STA are uncorrelated. In this report the effect of the correlation of the STA inputs on $\sqrt{V_b}$ is determined. Computer simulation results are included to check the calculations.

II. REVIEW

Figure one is a block diagram of the system being considered. The probability distribution, mean, variance and range of the data at each point have been derived by Penn and Trudell (reference (2)) and are

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included here for convenience. These numbers are derived as follows:

Assuming the $x_i(t)$ are uncorrelated, z has the binomial distribution (reference (3)).

$$P_z(Z) = \binom{M}{Z} p^Z q^{M-Z}$$

where $p = \Pr(x_i \geq 0) = 1/2$ for noise only.

Then

$$\bar{z} = Mp - \frac{1}{2} M = 0$$

$$\bar{V}_z^2 = Mp(1-p) = \frac{M}{4}$$

Now, assuming M is sufficiently large so that $p_z(Z)$ may be considered Gaussian

$$\bar{W} = \sqrt{\frac{2}{\pi}} \bar{V}_z = \sqrt{\frac{M}{2\pi}}$$

and

see appendix A.

$$\bar{V}_w^2 = \frac{\pi-2}{\pi} \bar{V}_z^2 = \frac{\pi-2}{\pi} \frac{M}{4}$$

If the space averager implements the equation

$$a = 4B_i - (B_{i-1} + B_{i-2} + B_{i+1} + B_{i+2})$$

we see that

$$\bar{V}_a^2 = 16 \bar{B}_i^2 - 8 \bar{B}_i \sum_i B_i + \sum_i \sum_j \bar{B}_i B_j$$

and if the beams are uncorrelated

$$\bar{V}_a^2 = 16 \bar{V}_w^2 + 4 \bar{V}_w^2 = 20 \bar{V}_w^2$$

Finally, if the a_i are uncorrelated

$$\sigma_b^2 = K \sigma_a^2 = 5K \frac{\pi-2}{\pi} M. \quad (1)$$

Where K is the number of samples.

If (-1, +1) clippers are used, rather than (0, 1) clippers as we have assumed equation (1) should be multiplied by 4 to find σ_b^2 .

The analysis has assumed:

1. Noise only at the hydrophones.
2. The noises at the hydrophones are uncorrelated.
3. M is sufficiently large so that $p_z(Z)$ is Gaussian.
4. The beams are uncorrelated.
5. The a_i are uncorrelated.

Assumptions 1 and 3 are reasonable; assumption 2 is unrealistic for small arrays and low frequency bands; assumption 4 is not true since the beams are correlated to some degree. Assumption 5 is the problem we now consider. Penn and Trudell also consider the problem of correlated samples, but from a different viewpoint.

III. EFFECT OF CORRELATION

so

$$b = \sum_{i=1}^K a_i$$

Then

$$\sigma_b^2 = \overline{b^2} - (\overline{b})^2 = \left[\sum_{i=1}^K a_i \right]^2$$

$$\sigma_b^2 = \sum_{i=1}^K \sum_{j=1}^K \overline{a_i a_j}$$

but

$$\overline{a_i a_i} = \overline{a_i^2} = \sigma_a^2$$

and

$$\overline{a_i a_j} = \overline{a_j a_i} \text{ since } \rho_a(\tau) \text{ is even.}$$

Thus

$$\begin{aligned} \sigma_b^2 &= K \sigma_a^2 + 2 \sigma_a^2 \sum_{i=1}^{K-1} \sum_{j=i+1}^K \rho_a[(j-i)\Delta T] \\ &= K \sigma_a^2 + 2 \sigma_a^2 \sum_{i=1}^{K-1} (K-i) \rho_a(i\Delta T) \quad (2) \end{aligned}$$

where $\rho_a(T)$ is the normalized autocorrelation function of a , and ΔT is the sample interval.

Thus, the problem is reduced to finding $\rho_a(t)$ in terms of $\rho_x(t)$, the input autocorrelation.

Assuming the beams are uncorrelated

$$\rho_a(t) = \rho_w(t),$$

so we must find $\rho_w(t)$ in terms of $\rho_z(t)$.

Papoulis shows (reference (4), p. 482), that if z is a Gaussian random variable with 0 mean, variance σ_z^2 and autocorrelation function $\rho_z(t)$ that

$$R_w(\tau) = \frac{2 \sigma_z^2}{\pi} \left[\cos \alpha + \alpha \sin \alpha \right]$$

where

$$\alpha = \sin^{-1} \rho_z(\tau).$$

Now if

$$\sin \alpha = \rho_z(\tau) \text{ then } \cos \alpha = [1 - \rho_z^2(\tau)]^{\frac{1}{2}}$$

so

$$R_w(\tau) = \frac{2V_z^2}{\pi} \left\{ [1 - \rho_z^2(\tau)]^{\frac{1}{2}} + \rho_z(\tau) \sin^{-1}[\rho_z(\tau)] \right\}.$$

Normalizing $R_w(\tau)$ we obtain

$$\begin{aligned} \rho_w(\tau) &= \frac{R_w(\tau) - (\bar{w})^2}{V_w^2} = \frac{R_w(\tau) - \frac{2}{\pi} V_z^2}{\frac{\pi-2}{\pi} V_z^2} \\ &= \frac{2}{\pi-2} \left\{ [1 - \rho_z^2(\tau)]^{\frac{1}{2}} + \rho_z(\tau) \sin^{-1}[\rho_z(\tau)] - 1 \right\}. \end{aligned}$$

Finally we note

$$\rho_z(\tau) = \frac{2}{\pi} \sin^{-1}[\rho_x(\tau)]$$

if the x_i are Gaussian variables with identical $\rho_{x_i}(\tau)$.

Thus our final result is

$$\begin{aligned} \rho_a(\tau) &= \frac{2}{\pi-2} \left\{ [1 - (\frac{2}{\pi} \sin^{-1} \rho_x(\tau))^2]^{\frac{1}{2}} + \frac{2}{\pi} \sin^{-1} \rho_x(\tau) \times \right. \\ &\quad \left. \times [\sin^{-1}(\frac{2}{\pi} \sin^{-1} \rho_x(\tau))] - 1 \right\} \end{aligned} \quad (3)$$

where we notice, if $\rho_x(\tau) \leq .5$, that

$$\rho_a(\tau) \approx \frac{2}{\pi-2} \left\{ [1 - \frac{4}{\pi^2} \rho_x^2(\tau)]^{\frac{1}{2}} + \frac{4}{\pi} \rho_x^2(\tau) - 1 \right\}$$

Equation 2 has been derived under assumptions 1, 2, 3 and 4 of Section II, and in addition we now require the $x_i(t)$ to be samples from a Gaussian process with identical autocorrelation functions.

IV. EVALUATION FOR A SPECIFIC CASE

It now remains to evaluate the effect of all this for a specific case of interest. Let the power spectral density of x , $S_x(f)$, be given by

$$S_x(f) = \frac{V_x^2}{2B} \quad \text{for } f_0 - \frac{B}{2} < |f| < f_0 + \frac{B}{2}$$

$$= 0 \text{ elsewhere.}$$

then

$$\rho_x(\tau) = \frac{\sin(\pi B \tau)}{\pi B \tau} \cos(2\pi f_0 \tau)$$

Appendix B lists the values of $\rho_x(\Delta T)$ for $\Delta T = 40 \mu\text{sec}$, $f_0 = 2500 \text{ Hz.}$, $B = 3000 \text{ Hz.}$ Now if $M = 48$, $K = 512$ we obtain, for the a_i uncorrelated

$$V_b = \sqrt{5 \frac{1.14}{3.14} (48)(512)} = 211 \text{ from equation 1.}$$

Then using the computed values of the correlation from appendix B, equation 2 and equation 3 we find

$$V_b = 211 \left[1 + \frac{2}{K} \sum_{i=1}^{K-1} \rho_a(i\Delta T) \right]^{\frac{1}{2}}$$

$$= 211 \left[1 + \frac{2}{512} (290) \right]^{\frac{1}{2}} = 211 (1.45) = 307$$

Thus the false assumption that the a_i are uncorrelated causes an error of roughly 50% in the computed value of the STA output standard deviation.

V. COMPUTER SIMULATION RESULTS

R. L. Gordon's DIMUS simulation program (reference (5)) was used to check the calculations. The program was run using uncorrelated, Gaussian noise at the inputs having a +3 db/ octave slope in the 1-4 band. No space averaging was used in order to eliminate the effects of beam to beam correlation. Thus, the program conforms to assumption 1, 2, 3 and 4 of Section II.

The program used (-1, +1) clippers and $K = 128$.

Using these parameters, equation 1 predicts

$$V_b = \left[(128) \frac{1.14}{3.14} (48) \right]^{\frac{1}{2}} = 47.22$$

if the STA inputs are uncorrelated. From equation 2 and equation 3

$$V_b = 47.22 \left[1 + \frac{2}{128} (83.73) \right]^{\frac{1}{2}} = 71.74$$

is the result obtained when the correlation is accounted for. This result was computed using the first 20 values of $\rho_x(\Delta T)$ for the +3 db/ octave noise used in the program.

Table one shows the standard deviation of the STA output for all 48 beams as computed from the simulation program. These standard deviations were each computed from 129 data points. The average of this data is 75.57 and the data has a standard deviation of 7.8. This number is close to the result obtained from equation 2, and we conclude that the computer simulation verifies the result of the analysis.

VI. CONCLUSIONS

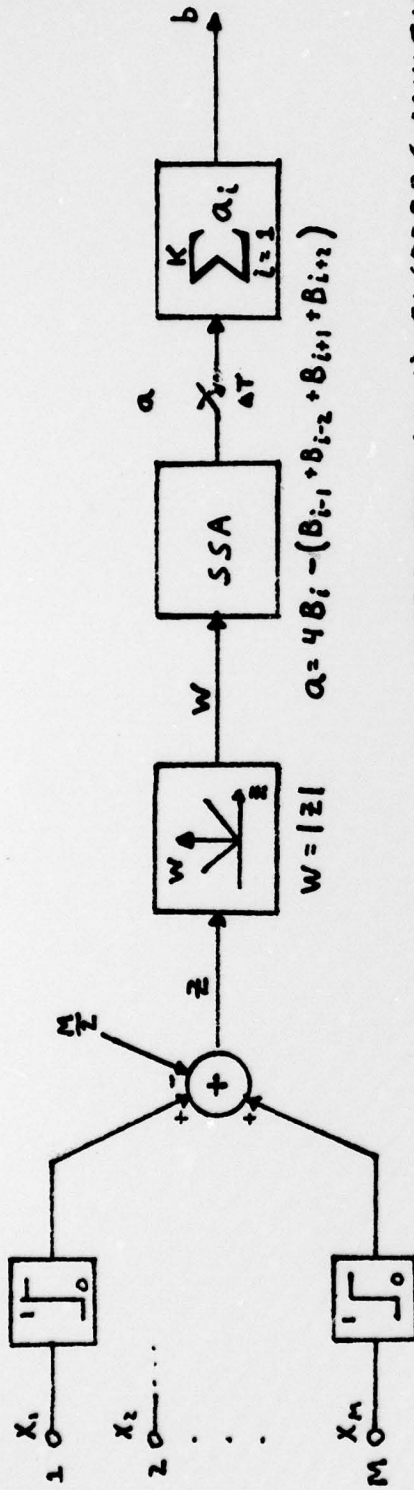
→ It has been shown that the numerical value to use for the standard deviation of the STA output is roughly 1.4 times the number calculated →

↙ assuming uncorrelated samples, if assumptions 1-4 are true. Computer simulation results verify the analysis. The effect of beam to beam correlation and of correlated noise at the hydrophones on the STA output standard deviation has not yet been determined, and is the subject of further study. ↗

VII. ACKNOWLEDGMENT

I wish to thank Mr. R. L. Gordon for setting up his DIMUS program to check these computations, and Mr. A. F. Magaraci for his kindly criticisms.

John P. Ianniello
J. P. IANNIELLO



$$a = 4B_i - (B_{i-1} + B_{i-2} + B_{i+1} + B_{i+2})$$

$$w = |z|$$

NOTE: FOR (-1,1) CLIPPERS MULTIPLY ∇_i^2 BY 4 AND RANGE BY 2.

$P_i(\cdot)$	z	w	a	b
$E(\cdot)$				
∇_i^2	$\frac{M}{4}$	$\sqrt{\frac{M}{2\pi}}$	$5 \frac{\pi-2}{\pi} M$	$5 K \frac{\pi-2}{\pi} M$
RANGE	$-\frac{M}{2} < z < \frac{M}{2}$	$0 < w < \frac{M}{2}$	$-2M < a < 2M$	$-2KM < b < 2KM$

TABLE ONE

<u>BEAM</u>	<u>STANDARD DEVIATION OF THE STA OUTPUT</u>
1	88.75
2	67.84
3	69.47
4	71.97
5	71.42
6	78.37
7	68.69
8	75.55
9	75.88
10	63.98
11	73.03
12	69.63
13	88.75
14	80.15
15	70.89
16	71.94
17	73.28
18	79.33
19	61.79
20	89.80
21	97.05
22	75.94
23	67.50
24	73.14
25	76.92
26	67.02
27	69.75
28	78.78
29	68.22
30	65.40
31	83.25
32	77.83
33	83.44
34	81.15
35	78.32
36	71.84
37	65.94
38	66.12

TABLE ONE (CONTINUATION)

<u>BEAM</u>	<u>STANDARD DEVIATION OF THE STA OUTPUT</u>
39	74.87
40	72.18
41	70.60
42	76.48
43	81.00
44	88.79
45	86.25
46	71.81
47	78.74
48	88.67

APPENDIX A

Derivation of $\bar{\omega}$ and σ_{ω}^2

Assume z is a Gaussian random variable with zero mean and variance σ_z^2 , then if

$$\omega = |z|$$

$$P_{rob}[\omega \leq W] = 2 P_{rob}[z \leq W] = 2 \int_0^W p_z(z) dz$$

then

$$\begin{aligned} p_{\omega}(\omega) &= \frac{d}{d\omega} P_{rob}[\omega \leq W] = 2 p_z(W) \\ &= \frac{2}{\sqrt{2\pi} \sigma_z} e^{-\frac{\omega^2}{2\sigma_z^2}} \quad \omega \geq 0. \end{aligned}$$

Now we find $\bar{\omega}$,

$$\bar{\omega} = \int_{-\infty}^{\infty} \omega p_{\omega}(\omega) d\omega = \int_0^{\infty} \sqrt{\frac{2}{\pi}} \frac{\omega}{\sigma_z} e^{-\frac{\omega^2}{2\sigma_z^2}} d\omega$$

let

$$t = \frac{\omega^2}{2\sigma_z^2}$$

then

$$dt = \frac{1}{\sigma_z^2} \omega d\omega \quad \text{and} \quad \omega d\omega = \sigma_z^2 dt$$

so

$$\bar{\omega} = \sqrt{\frac{2}{\pi}} \frac{1}{\sigma_z} \int_0^{\infty} \sigma_z^2 e^{-t} dt = \sqrt{\frac{2}{\pi}} \sigma_z [-e^{-t}]_0^{\infty} = \sqrt{\frac{2}{\pi}} \sigma_z.$$

Next, we find ∇_{ω}^2 , where

$$\nabla_{\omega}^2 = \overline{\omega^2} - (\overline{\omega})^2.$$

Now

$$\overline{\omega^2} = \int_0^{\infty} \sqrt{\frac{2}{\pi}} \frac{\omega^2}{\sqrt{z}} e^{-\frac{\omega^2}{2\sqrt{z}}} d\omega$$

let

$$t = \frac{\omega}{\sqrt{2}\sqrt{z}} \quad \text{then}$$

$$d\omega = \sqrt{2}\sqrt{z} dt \quad \text{and} \quad \omega^2 = 2\sqrt{z}^2 t^2$$

so

$$\omega^2 d\omega = 2\sqrt{2}\sqrt{z}^3 t^2 dt$$

and

$$\overline{\omega^2} = \frac{4\sqrt{z}^2}{\sqrt{\pi}} \int_0^{\infty} t^2 e^{-t^2} dt$$

but the integral is equal to

$$\frac{\sqrt{\pi}}{4} (1)$$

so $\overline{\omega^2} = \sqrt{z}^2.$

Therefore

$$\begin{aligned} \nabla_{\omega}^2 &= \overline{\omega^2} - (\overline{\omega})^2 = \sqrt{z}^2 - \frac{2}{\pi} \sqrt{z}^2 \\ &= \frac{\pi-2}{\pi} \sqrt{z}^2. \end{aligned}$$

¹Reference Data for Radio Engineers, ITT, 4th edition, p. 1079, eqv. 144.

APPENDIX B

$$P(\tau) = \frac{\sin \pi B \tau}{\pi B \tau} \cos(2\pi f_0 \tau)$$

$$P(40 \mu\text{sec}) = \frac{\sin(\pi(3 \times 10^3)(.04 \times 10^{-3}))}{\pi(3 \times 10^3)(.04 \times 10^{-3})} \cos(2\pi(2.5)(.04))$$

$$= .79$$

$$P(80) = .281$$

$$P(120) = -.247$$

$$P(160) = -.536$$

$$P(200) = -.505$$

$$P(240) = -.276$$

$$P(280) = .056$$

$$P(320) = .012$$

$$P(360) = -.06$$

$$P(400) = .116$$

$$P(440) = -.144$$

$$P(480) = -.063$$

$$P(520) = -.063$$

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